Chaotic atomic population oscillations between two coupled Bose-Einstein condensates with time-dependent asymmetric trap potential

Chaohong Lee, Lei Shi, Xiwen Zhu, Kelin Gao

State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics,

Wuhan Institute of Physics and Mathematics, The Chinese Academy of Sciences,

Wuhan, 430071, P. R. China

Wenhua Hai, Yiwu Duan

Department of Physics, Hunan Normal University, Changsha, 410081, P. R. China

Wing-Ki Liu

Department of Physics, University of Waterloo, Waterloo, Ontario, N2L3G1, Canada

(February 1, 2008)

Abstract

We have investigated the chaotic atomic population oscillations between two coupled Bose-Einstein condensates (BEC) with time-dependent asymmetric trap potential. In the perturbative regime, the population oscillations can be described by the Duffing equation, and the chaotic oscillations near the separatrix solution are analyzed. The sufficient-necessary conditions for stable oscillations depend on the physical parameters and initial conditions sensitively. The first-order necessary condition indicates that the Melnikov function is equal to zero, so the stable oscillations are Melnikov chaotic. For the ordinary parameters and initial conditions, the chaotic dynamics is simulated with numerical calculation. If the damping is absent, with the increasing of

the trap asymmetry, the regular oscillations become chaotic gradually, the corresponding stroboscopic Poincare sections (SPS) vary from a single island to more islands, and then the chaotic sea. For the completely chaotic oscillations, the long-term localization disappears and the short-term localization can be changed from one of the BECs to the other through the route of Rabi oscillation. When there exists damping, the stationary chaos disappears, the transient chaos is a common phenomenon before regular stable frequency locked oscillations. And proper damping can keep localization long-lived.

PACS numbers: 05.30.Jp, 03.75 Fi

Typeset using REVTEX

I. INTRODUCTION

The study of Bose-Einstein condensate (BEC) in gases can give new understanding of atomic, condensed-matter, and statistical physics. Due to the remarkable development in the ability to control atomic motion by optical techniques, BEC was first produced in a weakly interacting gas of alkali metal atoms held in magnetic trap in 1995 [1]. Following the first observation, many important experiments were carried out. The group of Durfee and Ketterle detected the condensate in cooled gases, the collective excitations, the collisions between separately-prepared condensates, and the pulsed output of a prototype atom laser [2,3], they also observed the interference between two Bose condensates and demonstrated that Bose condensed atoms are "laser-like", that is, they are coherent and show long-range correlations [4]. These results have direct implications for the atom laser and the Josephson effect for atoms. Recently, solitons have been generated in BEC by properly phase imprinting [5], the phase of a BEC wave-function was prepared with optical imprinting techniques and measured with a Mach-Zehnder matter-wave interferometer that makes use of optically induced Bragg diffraction.

Experimental achievements of BEC caused great theoretical interests in this novel field. There exists abundant nonlinear dynamics in BEC, for the macroscopic condensate wave function obeys a nonlinear equation, which is called as the Gross-Pitaveskii equation (GPE) [6]. Vortex stability in BEC is explained by using a two-mode model [7]. Oscillations of atomic populations and collective excitations at high energies was detailed [8], the complex dependencies of these excitation energies bring us close to the notion of chaos, and the role of chaotic motions in the dynamics of BEC remains to be studied. Large amplitude oscillations of condensed trapped atoms to external driving magnetic fields was analyzed by the group of Smerzi [9], their results of frequencies and excitation times of collective oscillations consistent with the experimental data very much. An appropriate semiclassical limit for GPE with an additional chaotic potential was given out by using a semiclassical interpretation of the Wigner function [10]. Assuming the background density and velocity vary slowly on the

soliton scale, Busch derived the equation of motion for dark soliton propagating through an effectively one-dimensional cloud of BEC, by using a multiple scale boundary layer theory. Recently, Filho et al. investigated the dynamics of the growing and collapsing of BEC in a system of trapped ultracold atoms with negative scattering lengths and found that the number of atoms can go far beyond the static stability limit [12].

Current efforts are being focused on coupled two-component and multi-component BEC. Interference and dynamics of component separation in two-component BEC was observed [4,13]. The quantum statistics of the ground state of a two-mode model for coupled BECs was analyzed [14], and strong squeezing of the number difference for positive nonlinearities and a regime of squeezing in the relative phase for negative nonlinearities were revealed. The dynamics of Josephson-like oscillations between two coupled BECs was studied using the time-dependent variational method [15], from the calculation result, the tunneling dynamics is "coherent" when the trap is not displaced, that is, the orbitals of each condensate do not change; on the other hand, the change in the condensate orbitals has a strong influence in the tunneling dynamics when the trap is displaced. Smerzi and Raghavan researched the coherent atomic tunneling and oscillations between two zero-temperature BECs confined in a double-well magnetic trap in the case of weakly Josephson coupling [16,17]. The coupling was provided by a laser barrier in a double well magnetic trap or by Raman coupling between two condensates in different hyperfine levels. The dynamics of phase difference and fractional population imbalance was described with the two-mode nonlinear GPE called as Bose Josephson junction (BJJ) equation. In addition to the nonsinusoidal anharmonic generalization of the ac Josephson effect and plasma oscillations occurring in the superconductor junction (SJJ), the macroscopic quantum self-trapping (MQST, a self-maintained population imbalance with nonzero average value of the fractional population imbalance.) and the $\pi-$phase oscillations (the time averaged value of the phase difference is equal to π) were also observed.

In the case of time-dependent trapping potential and non-negative damping and finite temperature effects, the more interesting nonlinear dynamics emerges out, such as chaos.

Abdullaev and Kraenkel analyzed the coherent atomic oscillations and resonances between two coupled BECs in a double-well trap with time-dependent tunneling amplitude for different damping [18]. With a slowly varying trap, the nonlinear resonances and chaos exist in the oscillations of the fractional imbalance. The conditions for chaotic macroscopic quantum tunneling phenomena were obtained with the use of the Melnikov function approach, and the chaotic oscillations depend on the frequency and modulation amplitude sensitively. For the rapidly varying case, the averaged system was given out by using the multiscale expansion method. They also considered the macroscopic quantum tunneling and resonances in coupled BECs with oscillating atomic scattering length [19]. The chaotic oscillations in the relative atomic population due to the overlaps between nonlinear resonances were showed. And the possibility of stabilization of the unstable -mode regime was derived from the analyzing of the oscillations in the rapidly varying case.

We know the laser barrier position and the laser beam intensity of the laser beam in the trap can be modified in experiments, so the trap asymmetry and the amplitude of the tunneling between the coupled BECs can be time-dependent. In our present paper, we will analyze the chaotic oscillation of the fractional population imbalance between two Josephson coupled BECs with time-dependent asymmetric trap potential, using both analytical and numerical approach. The structure of this paper is as follows. In this section, we briefly review both experimental and theoretical developing of BEC, and also show the purpose of the paper. The effective particle model for the oscillations of the fractional population imbalance is derived from the two-mode GPE in the next section. In the case of small time-dependent trap asymmetry and small damping, using the analytical method [20,21] developed by us, the chaotic dynamics of the fractional imbalance near the separatrix solution is analyzed in details in the third section. The conditions for chaotic oscillation and criteria for the onset of chaos are also obtained. And the regions of regular and chaotic oscillation are showed. In the fourth section, the population oscillations are simulated with numerical approach for arbitrary time-dependent trap asymmetry and damping. In the last section, a brief precise summary and discussion are given out.

II. THE EFFECTIVE PARTICLE MODEL FOR THE OSCILLATIONS OF THE FRACTIONAL POPULATION IMBALANCE

Ignoring the damping and finite-temperature effects, the problem of coupled Bose-Einstein condensates in a double well trap can be described with the following nonlinear two-mode dynamical equations

$$
i\hbar\frac{\partial\psi_1}{\partial t} = [E_1 + U_1 |\psi_1|^2]\psi_1 - K\psi_2,
$$
$$
i\hbar\frac{\partial\psi_2}{\partial t} = [E_2 + U_2 |\psi_2|^2]\psi_2 - K\psi_1. \tag{1}
$$

Where, E_1 and E_2 are zero-point energies in each well, $U_1 |\psi_1|^2$ and $U_2 |\psi_2|^2$ are proportional to the atomic self-interacting energies, and K describes the amplitude of the tunneling between two condensates. These parameters are defined by the overlap integrals of the time-dependent Gross-Pitaevsky eigenfunctions. The above equations are named as BJJ equations, which have been derived from GPE in reference [17].

The wave function ψ_i can be written in the form of $\psi_i = \sqrt{N_i}\exp(i\theta_i)$, here, N_i and θ_i are the amplitudes for general occupations and phases respectively. Then the fractional population imbalance can be defined as

$$
z(t) = \frac{N_1(t) - N_2(t)}{N_T} = \frac{|\psi_1|^2 - |\psi_2|^2}{|\psi_1|^2 + |\psi_2|^2}. \tag{2}
$$

here, $N_T = N_1(t) + N_2(t)$ is a constant total number of condensed atoms. And the relative phase is

$$
\phi(t) = \theta_2(t) - \theta_1(t). \tag{3}
$$

Then the fractional population imbalance and the relative phase obey the following differential equations

$$
\frac{dz}{dt} = -\frac{2K}{\hbar}\sqrt{1-z^2}\sin\phi,
$$
$$
\frac{d\phi}{dt} = \frac{2K}{\hbar}(\Delta E + \Lambda z + \frac{z}{\sqrt{1-z^2}}\cos\phi). \tag{4}
$$

The parameters ΔE and Λ determine the dynamic regimes of the BEC atomic tunneling and they can be expressed as

$$\Delta E = \frac{E_1 - E_2}{2K} + \frac{U_1 - U_2}{4K} N_T. \tag{5}$$

$$\Lambda = U N_T / 2K \quad , \quad U = (U_1 + U_2)/2. \tag{6}$$

Because of the overlapping condensate, there exists different kind of damping for different type of overlapping. For examples, if we take into account a noncoherent dissipative current of normal-state atoms, the differential equation which describes the oscillations of the atomic population including the damping term $-\eta d\phi/dt$; and the damping has the form $-\eta z(t)$ for the two interacting condensates with different hyperfine levels in a single harmonic trap. In the case of time-independent parameter K, we can rescale $(\frac{2K}{\hbar})t$ to a dimensionless time t. The motion of the fractional population imbalance and relative phase is very similar to a nonrigid pendulum. The Hamiltonian of the unperturbed $(\Delta E = cons\tan t, \eta = 0)$ dimensionless system is as follows

$$H = \frac{\Lambda z^2}{2} + \Delta E z - \sqrt{1 - z^2} \cos \phi. \tag{7}$$

The corresponding canonical equations of the motion are equivalent to the equations of the oscillations of the atomic population imbalance and relative phase, their forms are

$$\frac{d\phi}{dt} = \frac{\partial H}{\partial z} \quad , \quad \frac{dz}{dt} = -\frac{\partial H}{\partial \phi}. \tag{8}$$

For the time-independent trapping potential, the energy of the above system is conservative.

In order to see the oscillations of the fractional population imbalance more transparently, we introduce an effective classical particle whose coordinate is z, moving in a potential V with the initial energy $E_{eff}^0 = [\dot{z}(t)|_{t=t_0}]^2/2 + V|_{t=t_0}$. If the trapping potential is time-independent, the effective potential V is time-independent too, and the motion of the effective particle is regular; otherwise, for the time-dependent trapping potential, V varies with time and the corresponding motion is anharmonic and even chaotic. The chaotic dynamics in atomic

tunneling will be detailed in the following sections. For the time-independent constant coupling K, the Hamiltonian of the effective particle is given by

$$H_{eff} = \frac{1}{2}p_z^2 + V = \frac{1}{2}(1 - H^2)$$
$$V = \frac{1}{2}z^2(1 - \Lambda H + \frac{1}{4}\Lambda^2 z^2) + (\frac{1}{2}\Delta E \Lambda z^3 + \frac{1}{2}\Delta E^2 z^2 - \Delta E H z). \tag{9}$$

The Hamiltonian's canonical equations of the motion are

$$\frac{dz}{dt} = \frac{\partial H_{eff}}{\partial p_z} \quad , \quad \frac{dp_z}{dt} = -\frac{\partial H_{eff}}{\partial z}. \tag{10}$$

For the symmetric trapping potential ($\Delta E = 0$), the effective potential V is time-independent, increasing the value of $(1 - \Lambda H)$ from negative to positive changes the effective potential from a double-well to a parabolic. The effective particle moves between the classical turning points, where the kinetic energy of the effective particle is zero. Fig. 1 shows the changing of the shape of the effective potential, (A) for different values of H with fixed value of Λ, and (B) for different values of Λ with fixed value of H.

The motion in the parabolic potential is Rabi oscillation with a zero time-average value of the fractional population imbalance z. For fixed parameters Λ and H, the oscillations with small effective energies H_{eff} are sinusoidal, the increasing of the effective energies adds higher harmonics to the sinusoidal oscillations.

In the case of double-well potential, the motion is very different from the case of the parabolic potential. If the effective energies is greater than the barrier between two wells, that is, $H_{eff} > 0$, the motion is a nonlinear Rabi oscillation with a zero time-average value of z, it corresponds to the periodic flux of atoms from one BEC to the other. If the effective energy is little than the potential barrier, $H_{eff} < 0$, the particle is confined in one of the two wells, it means the localization of atomic population in one of the two condensates, and this localizing phenomenon has been named as macroscopic quantum self-trapping (MQST). At the threshold point, the effective energy is equal to the potential barrier, $H_{eff} = 0$, the corresponds threshold motion separating the above two regimes, the separatrix solution for the right-hand side well is

$$z_s(t) = 2\sqrt{(\Lambda H - 1)/\Lambda^2}\, \sec h\xi \quad, \quad \xi = C_0 + t\sqrt{\Lambda H - 1}. \tag{11}$$

Where, constant C_0 is determined by the initial conditions. Considering the physical constrain, the amplitude of the fractional atomic population imbalance oscillations must be little than one, i.e.,$|z|_{\max} \leqslant 1$, so the abstract value z of at points with the lowest potential energy must be little than one too, this requires the parameters satisfy $\sqrt{2(\Lambda H - 1)/\Lambda^2} \leqslant 1$, the atoms completely localize on one of the two condensates when $\sqrt{2(\Lambda H - 1)/\Lambda^2} = 1$. And if the amplitude of the separatrix solution is lager than one, i.e., $2\sqrt{(\Lambda H - 1)/\Lambda^2} > 1$, then there only exists MQST.

Based upon the above analysis, we know that the pitchfork bifurcation occurs at the point with $\Lambda H = 1$ for the time-independent symmetric trapping potential, that is to say the equilibrium point at the origin changes stability type and two new additional equilibrium points are created. For the asymmetric and time-dependent trapping potential, the bifurcation becomes more complex, the regular oscillations become chaotic through the route of period doubling.

III. CHAOTIC OSCILLATIONS NEAR THE SEPARATRIX SOLUTION WITH SMALL TRAP ASYMMETRY

It is very interesting to investigate the dynamics of the fractional population imbalance near the separatrix solution, that is, the initial conditions and physical parameters are very close to the separatrix of the unperturbed symmetric system. In the case of small trap asymmetry ΔE and small damping η, they can be looked as perturbations to the symmetric system, from the general theory of nonlinear driven oscillations, the chaotic macroscopic quantum tunneling phenomena appears when the trap asymmetry is time-dependent. For the two interacting condensates with different hyperfine levels in a single harmonic trap, the damping has the form $-\eta dz/dt$, driving from the effective potential in the previous section, the Newtonian equation of the motion for the fractional population imbalance is given as the following Duffing equation.

$$\frac{d^2z}{dt^2} - (\Lambda H - 1)z + \frac{\Lambda^2}{2}z^3 = -\frac{3}{2}\Delta E \Lambda z^2 - \Delta E^2 z + \Delta E H - \eta \frac{dz}{dt}. \tag{12}$$

In addition to a time-independent trap asymmetry ΔE_0, we impose a sinusoidal variation so that we can write the asymmetry term as $\Delta E = \Delta E_0 + \Delta E_1 \sin \omega t$. When the intensity of the laser beam is fixed, varying the laser barrier position can realize this. Writing the trap asmmetry and the damping as following form

$$\Delta E = \Delta E_0 + \Delta E_1 \sin \omega t = \varepsilon(F_0 + F_1 \sin \omega t) \quad , \quad \eta = \varepsilon\mu. \tag{13}$$

In the above, ε is a dimensionless parameter. Using the analytical approach developed by us [20,21], we write the solution close to the separatrix solution as the following expansions

$$z = \sum_{i=0}^{+\infty} \varepsilon^i z_i = z_0 + \varepsilon z_1 + \varepsilon^2 z_2 + \dots \tag{14}$$

Here, z_i are the $i-th$ order corrections. Substituting the above expression into the Newtonian equation of the motion, comparing the coefficient function of every ε^i of both sides of the differential equation, setting ε as 1, then we obtain z_i satisfy

$$\frac{d^2 z_0}{dt^2} - (\Lambda H - 1)z_0 + \frac{\Lambda^2}{2}z_0^3 = 0. \tag{15}$$

$$\frac{d^2 z_i}{dt^2} - (\Lambda H - 1)z_i + \frac{3\Lambda^2}{2}z_0^2 z_i = \epsilon_i,$$

$$\epsilon_1 = -\eta\frac{dz_0}{dt} + \Delta E H - \frac{3}{2}\Delta E \Lambda z_0^2, \epsilon_2 = -\eta\frac{dz_1}{dt} - 3\Delta E \Lambda z_0 z_1 - \frac{3}{2}\Lambda^2 z_0 z_1^2, \dots \tag{16}$$

The zero-order solution is the separatrix solution, and the basic solutions of the high-order corrections are as follows

$$z_{i1}^0 = \frac{dz_0}{dt} = -\frac{2(\Lambda H - 1)}{\sqrt{\Lambda^2}} \sec h\xi \tanh \xi. \tag{17}$$

$$z_{i2}^0 = z_{i1}^0 \int (z_{i1}^0)^{-2} dt$$

$$= \frac{-\sqrt{\Lambda^2}}{16(\Lambda H - 1)^{3/2}} \sec h^2\xi(\cosh 3\xi - 9\cosh \xi + 12\xi \sinh \xi). \tag{18}$$

10

So the general expressions of $i - th$ corrections are in form of

$$z_i = z_{i2}^0 \int\limits_{C_1}^t z_{i1}^0 \epsilon_i dt - z_{i1}^0 \int\limits_{C_2}^t z_{i2}^0 \epsilon_i dt. \tag{19}$$

Constants C_1 and C_2 are determined by the initial conditions and physical parameters. Apparently, $|z_{i1}^0| \to 0$ and $|z_{i2}^0| \to +\infty$, when time $t \to \pm\infty$. Solving the $i - th$ order equations one by one, we can obtain ϵ_i are time-periodic functions with finite amplitudes. This means the high-order corrections are non-convergent unless the coefficient functions of the growing function z_{i2}^0 are equal to zero. So the general motion is unstable periodic oscillations, the necessary-sufficient conditions for stable oscillations are

$$\lim_{t\to\pm\infty} \int\limits_{C_1}^t z_{i1}^0 \epsilon_i dt = 0. \tag{20}$$

The above conditions are non-integrable, clearly, they contain the following necessary conditions

$$\int\limits_{-\infty}^{+\infty} z_{i1}^0 \epsilon_i dt = 0. \tag{21}$$

Apparently, the first integral $(i = 1)$ of the necessary conditions is the Melnikov function of the system, the necessary conditions indicate that the Melnikov function is equal to zero; this means that the stable oscillations are Melnikov chaotic. But because of the non-sufficient property of the above condition, not all chaotic oscillations are stable. Integrating the above equations, one can obtain the necessary conditions are a series of relations of the initial conditions and parameters, for fixed initial conditions, modifying the parameters can control the instability of the chaotic oscillations. Substituting the expressions of z_{11}^0 and ϵ_1 into the necessary condition, integrating it yields the first-order condition

$$-\frac{8\eta(\Lambda H - 1)\sqrt{\Lambda H - 1}}{3\Lambda^2} - \frac{2\Delta E_1 H}{\sqrt{\Lambda^2}}\omega\pi\cos\frac{\omega C_0}{\sqrt{\Lambda H - 1}}\sec h\frac{\omega\pi}{2\sqrt{\Lambda H - 1}}$$

$$+\frac{2\Delta E_1(\Lambda H - 1)\sqrt{\Lambda H - 1}}{\Lambda\sqrt{\Lambda^2}}(1 + \frac{\omega^2}{\Lambda H - 1})\omega\pi\cos\frac{\omega C_0}{\sqrt{\Lambda H - 1}}\sec h\frac{\omega\pi}{2\sqrt{\Lambda H - 1}} = 0. \tag{22}$$

11

The above necessary condition is irrelative to the time-independent trap asymmetry ΔE_0, this means the chaotic oscillations are caused by the time-dependent trap asymmetry, but it is not to say that the stability is irrelative to the time-independent trap asymmetry, actually, the sufficent-necessary conditions and high-order necessary conditions are relations of it and other parameters. For the same parameters, the distribution of stability curves sensitive depends on the initial conditions, to show explicitly this dependence we have chosen a series of value of the initial constant C_0, with the growing of the value of C_0, the curves become denser and denser, this illustrates the existence of chaos, see Fig. 2. The changing between the regular oscillations and the chaotic oscillations is showed in Fig.3. Regions above the curves correspond to chaotic oscillations of the fractional population imbalance and those below correspond to regular oscillations. There exist two chaotic regions separated by a special frequency which is determined by the physical parameters, and this frequency can cause an unstable nonlinear resonance. When the damping becomes stronger and stronger, the regions of chaotic oscillations become smaller and smaller, and the regular region becomes larger and larger.

IV. NUMERICAL SIMULATION

In general, the atomic population oscillation is far away from the separatrix solution, and then the oscillation dynamics can not be obtained from the previous analytical method. In this section, using the fourth Runge-Kutta method with variable step-width, the chaotic population oscillations are simulated by straightforward numerical integration of the motion equations of the dimensionless model of system (4) with constant parameter K, and the trap asymmetry is in form of $\Delta E = \Delta E_0 + \Delta E_1 \sin \omega t$, and the damping of the population oscillation is $-\eta z(t)$. In the time -independent symmetric trap ($\Delta E = 0$), because of the damping, both Rabi oscillation and MQST reach an equilibrium state with zero population imbalance, see Fig. 4 (B) and (F), for the time-independent asymmetric case, the equilibrium state departure from the zero population imbalance, see (D) and (H) of Fig.4. Ignoring the

damping effects, the oscillations are regular, they contain two different kinds, Rabi oscillation and MQST,see (A), (E), (C) and (G) of Fig.4.

For the time-dependent asymmetric trap potential, the chaotic oscillation emerges out. Sampling a single trajectory every period of the varying of the trap asymmetry, then we can obtain the stroboscopic Poincare section (SPS). When the damping is absent, with the increasing of the time-dependent trap asymmetry ΔE_1, the sections vary from a single island into a lot of islands, and at last all islands are submerged by the chaotic sea. This means the periodic oscillations become quasi-periodic, and then chaotic. Fig. 5 is the SPS of $(z, dz/dt)$, with $z(0) = 0.5$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 4\pi$ and $\eta = 0.0$, for these initial conditions if the damping and the trap asymmetry are absent, the corresponding oscillation is Rabi oscillation. When $\Delta E_1 = 3.000$, there is only a single island. Then it is separated into six islands when ΔE_1 increase to 6.000. For larger trap asymmetry, $\Delta E_1 = 6.750$, the regular islands are surrounded by the chaotic sea. For large enough trap asymmetry, $\Delta E_1 = 7.500$, the regular islands are all submerged by the chaotic sea, and the sea is symmetrical to $z = 0$. Starting from the MQST, the SPS with $z(0) = 0.75$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 2\pi$ and $\eta = 0.0$ is showed in Fig. 6, the similar dynamics is exhibited. For small ΔE_1 $(1.000, 1.560$ and $1.565)$, the time-averaged value of the fractional population imbalance is non-zero, the atoms are localized on one of the condensates. However, for large enough ΔE_1 (1.700), the chaotic sea is symmetrical to $z = 0$. This indicates that, in the completely chaotic oscillation, the time-averaged value of the fractional population imbalance is zero, and the long-lived MQST or localization disappears.

The completely chaotic oscillations of the fractional population imbalance from Rabi oscillation and MQST are presented in Fig. 7. The left column corresponds to $z(0) = 0.5$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 4\pi$, $\Delta E_1 = 7.500$ and $\eta = 0.0$, the right column corresponds to $z(0) = 0.75$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 2\pi$, $\Delta E_1 = 1.700$ and $\eta = 0.0$. The first row is the time evolution of z, the second row is the power spectra of the corresponding oscillation. Apparently, through the tunnel of Rabi oscillation, the short-term localization or MQST can be changed from one of the BECs to the other, and the

13

corresponding power spectra is continuous.

Because of the existence of the damping, the dimensionless system is not a Hamiltonian system but a dissipative system and the volume in phase space will decrease through time evolution. Factually, these effects are the basic reason for the complex oscillation behavior. A common phenomenon in these dynamical systems is that they seem to behave chaotically during some transient periods, but eventually fall onto periodic stable attractors. This has been called as the transient chaos or chaotic transient. Superlong transient chaos occurs commonly in dissipative dynamical system, in this case, oscillations starting from random initial conditions oscillate chaotically for a very long time before they set into the final attractors which are usually regular and stable [22,23]. In our system, we also find the transient chaos and final attractors. Using the SPS of $(z, dz/dt)$, we exhibit the attracting process of the transient chaos and the fixed points of the final attractors. The phase trajectories of the final attractors are also showed.

For a certain damping parameter η, and fixed value of parameter Λ, ΔE_0, there exist many types of attractors when ΔE_1 is changed. For the same ΔE_1, different initial conditions will lead different final states. Starting from Rabi oscillation, with $z(0) = 0.5$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 4\pi$ and $\eta = 0.01$, for different parameter ΔE_1, the SPS of the attracting processes and the final attractors, and the phase trajectories of the final attractors are presented in Fig. 8, the left column shows the SPS of the attracting processes, the right column shows the phase trajectories and SPS of the final states, (A) and (B) for $\Delta E_1 = 3.000$, (C) and (D) for $\Delta E_1 = 7.500$. In the SPS, after the transient chaos, the sampled points gradually come to the final fixed points. The phase trajectories of final oscillations are closed curves, and the corresponding SPS only contain fixed points which are noted as small circles, so the final oscillations are frequency-locked (FL). When $\Delta E_1 = 3.000$, there is only a single fixed point in the SPS, the corresponding final oscillation is a period-one limit-cycle with frequency ω; while for $\Delta E_1 = 7.500$, there exist five fixed points, and then the final oscillation is a $\frac{1}{5}$ FL motion, this means the oscillating frequency is $\frac{1}{5}\omega$. Fig. 9 presents the similar dynamics starting form MQST, with $z(0) = 0.75$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$,

14

$\omega = 2\pi$ and $\eta = 0.001$, for different parameter ΔE_1. Where, (A) and (B) for $\Delta E_1 = 1.000$, (C) and (D) for $\Delta E_1 = 1.700$. The transient chaos and the FL oscillations appear too. When $\Delta E_1 = 1.000$, the eventual oscillation is a period-one limit cycle with a non-zero time-averaged value of the fractional population imbalance z, so the atoms are localized on one of the condensates. Amazedly, for large ΔE_1 (1.700), due to the damping effects, the final $\frac{1}{6}$ FL oscillation possesses of a non-zero time-averaged value of z, comparing with the non-damping regime (Fig. 6), one can obtain that the proper damping can keep the MQST long-lived.

V. SUMMARY AND DISCUSSION

Using both analytical and numerical methods, we analyzed the chaotic oscillations between two coupled Bose-Einstein condensates with time-dependent asymmetric trap potential. The trap asymmetry has been chosen as $\Delta E = \Delta E_0 + \Delta E_1 \sin \omega t$, this can be realized by varying the laser barrier position of the laser beam which possesses of fixed value of intensity. The damping of the oscillations of the fractional population imbalance is in form of $-\eta z(t)$, it commonly exists in the two interacting condensates with different hyperfine levels in a single harmonic trap.

In the perturbative regime, the population oscillations have been depicted with Duffing equation, the chaotic oscillations near the separatrix solution are detailed. The form of the general solution and the sufficient-necessary conditions for stable oscillations are obtained. These conditions sensitively depend on the initial conditions and the physical parameters, and the first necessary condition indicates that the Melnikov function of system is equal to zero, so the stable oscillations are Melnikov chaotic. The stable curves are presented out for different initial conditions, the sensitive dependence exhibits implicitly in the figures. Varying the damping strength, the regions of chaotic oscillations and regular oscillations can be changed into the other.

However, the general oscillations are not close to the separatrix solution, and the usual

parameters are not in perturbative regime, in such case, the numerical method is very useful. Using the fourth Runge-Kutta method with variable step-width, the chaotic population oscillations are simulated by straightforward numerical integration of the dimensionless motion equations. When the damping disappears, with the increasing of ΔE_1, the regular oscillations gradually become chaotic, and in the completely chaotic regime, the long-lived localization or MQST disappears. In the SPS, the single regular island is separated into many little islands, and then all islands are submerged into the chaotic sea. In the completely chaotic oscillations, the long-term localization or MQST disappears and the short-term localization or MQST can be changed from one of the BECs to the other through the tunnel of the Rabi oscillation. When the damping exists, due to the damping effects, the system is not a Hamiltonian system but a dissipative one, and the volume of the phase space is reduced by time evolution. Then the stationary chaos disappears, the transient chaos is a common phenomenon before regular stable frequency locked oscillations. Surprisingly, the proper damping strength can keep the localization or MQST long-lived.

In experiments, the long-term average lifetime of the transient chaotic oscillation requires the measurements must be long-term too. So the prediction of the relation between the average lifetime of the transient and the physical parameters (η, ΔE_0, ΔE_1 and Λ) may be a practical problem. And if one want to observe the long-lived localization or MQST, the understanding of the basins of attraction of the eventual FL oscillations in the parameter space will give some useful indication of how to choose the physical parameters. We will report these results in other papers.

FIGURES

FIG. 1. The changing of the shape of the effective potential V, (A) with fixed $\Lambda = 2.0$ and different values of H, (B) with fixed $H = 0.5$ and different values of Λ.

FIG. 2. The stability curves for different initial conditions with $H = 0.5$, $\Lambda = 0.5$, $\Delta E_0 = 0.0$ and $\eta = 0.5$.

FIG. 3. The regions of chaotic oscillations for different values of the damping parameter η, with $\Lambda = 4.0$, $\Delta E_0 = 0$ and $H = 0.5$.

FIG. 4. The time evolution of the fractional population imbalance z with $\Lambda = 10$, the left column with initial conditions $z(0) = 0.5$ and $\phi(0) = 0.0$, the right column with $z(0) = 0.8$, $\phi(0) = 0.0$. (A) and (E) with $\Delta E = 0.0$ and $\eta = 0.0$; (B) and (F) with $\Delta E = 0.0$ and $\eta = 0.5$; (C) and (G) with $\Delta E = 1.0$ and $\eta = 0.0$; (D) and (H) with $\Delta E = 1.0$ and $\eta = 0.5$.

FIG. 5. The stroboscopic Poincare section (SPS) of $(z, dz/dt)$ with $z(0) = 0.5$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 4\pi$, $\eta = 0.0$ and different values of ΔE_1.

FIG. 6. The stroboscopic Poincare section (SPS) of $(z, dz/dt)$ with $z(0) = 0.75$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 2\pi$, $\eta = 0.0$ and different values of ΔE_1.

FIG. 7. The completely chaotic oscillations and the corresponding power spectra. The left column corresponds to $z(0) = 0.5$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 4\pi$, $\Delta E_1 = 7.500$ and $\eta = 0.0$. The right column corresponds to $z(0) = 0.75$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 2\pi$, $\Delta E_1 = 1.700$ and $\eta = 0.0$.

FIG. 8. The stroboscopic Poincare section (SPS) of $(z, dz/dt)$ and frequency-locked oscillations with $z(0) = 0.5$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 4\pi$, $\eta = 0.01$ and different values of ΔE_1. (A) and (B) with $\Delta E_1 = 3.000$, (C) and (D) with $\Delta E_1 = 7.500$.

FIG. 9. The stroboscopic Poincare section (SPS) of $(z, dz/dt)$ and frequency-locked oscillations with $z(0) = 0.75$, $\phi(0) = 0.0$, $\Delta E_0 = 0.0$, $\Lambda = 10.0$, $\omega = 2\pi$, $\eta = 0.001$ and different values of ΔE_1. (A) and (B) with $\Delta E_1 = 1.000$, (C) and (D) with $\Delta E_1 = 1.700$.

REFERENCES

[1] M. H. Anderson, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science 269, 198 (1995).

[2] K. B. Davis, M. -O. Mews, M. R. Anderson, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. 75, 3969 (1995).

[3] D. S. Durfee, and W. Ketterle, Optics Express 2, 299 (1998).

[4] M. R. Andrews, C. G. Tounsend, H. -J. Miesner, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science 275, 637 (1997).

[5] J. Denschlag, et al., Science 287, 97 (2000).

[6] L. P. Pitaevskii, Sov. Phys. JETP 13, 451 (1961); E. P. Gross, Nuovo Cimento 20, 454 (1961), J. Math. Phys. 4, 195 (1963).

[7] Victor M., et al., Phys. Rev. A 62, 033601(2000).

[8] L. You, R. Walsworth, and W. Hoston, Optics Express 1, 293 (1997).

[9] Augusto Smerzi and Stefano Fantoni, Phys. Rev. Lett. 78, 3589 (1997).

[10] S. A. Gardiner, et al., Phys. Rev. A 62, 023612 (2000).

[11] Th. Busch and J. R. Anglin, arXiv: xxx.lanl.gov, cond-matter/980948.

[12] D. S. Hall, et al., Phys. Rev. Lett. 81, 1539 (1998).

[13] Victo S. Filho, et al., Phys. Rev. A 62, 033605 (2000).

[14] M. J. Steel and M. J. Collett, Phys. Rev. A 57, 2920 (1998).

[15] Chi-Yong Lin and E. J. V. de Passos, Phys. Rev. A, 055603 (2000).

[16] A. Smerzi, S. Fantoni, S. Giovanzzi, and S. R. Shenoy, Phys. Rev. Lett. 79, 4950 (1997).

[17] S. Raghavan, A. Smerzi, S. Fantoni, and S. R. Shenoy, Phys. Rev. A 59, 620 (1999).

[18] F. Kh. Abdullaev and R. A. Kraenkel, Phys. Rev. A 62, 023613 (2000).

[19] F. Kh. Abdullaev and R. A. Kraenkel, arXiv: cond-mat/0005445 25 May 2000.

[20] Wenhua Hai, et al., Phys. Rev. A 61, 052105 (2000).

[21] Chaohong Lee, et al., Acta Physica Sinica 47, 1409 (1998).

[22] J. Hoffnagle and R. G. Brewer, Science 265, 213 (1994), Phys. Rev. Lett. 71, 1828 (1993), Phys. Rev. A 50, 4157 (1994).

[23] Jing-Ling Shen, Hua-Wei Yin, and Jian-Hua Dai, Phys. Rev. A 55, 2159 (1997).